Martley C of E Primary School

Calculation Policy for Mathematics

How we teach calculations.

2014

About our Calculation Policy

The following calculation policy has been devised to meet the requirements of the National Curriculum 2014 for the teaching and learning of maths, and is also designed to give pupils a smooth progression of learning in calculations across the school. In Reception the Early Learning Goals are followed in the progression of maths. This policy follows on from the Early Years Foundation Stage.

Age related expectations:

The calculation policy is organised according to the age related expectations set out in the National Curriculum 2014; however, **it is vital that pupils are taught according to the stage that they are currently working at,** **ONLY** being moved onto the next level when their subject knowledge is secure and they are able to move forward.

Providing a context for calculation:

It is important that any type of calculation is given a real life context or a problem solving approach to help build children's understanding of the purpose of the calculation, and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons.

Choosing a calculation method:

Children need to be taught and encouraged to use the following processes in deciding what approach they will take to a calculation, to ensure they select the most appropriate method for the numbers involved:

To work out a tricky calculation
Approximate
Calculate
Check it mate!

Can I do it in my head using a mental strategy?

Could I use some jottings to help me?

Should I use a written method to work it out?

PROGRESSION THROUGH CALCULATIONS FOR ADDITION

MENTAL CALCULATIONS
(ongoing)
These are a **selection** of mental calculation strategies:

Mental recall of number bonds
This includes number fans, number cards, ICT programs and other practical
equipment
6 + 4 = 10   □ + 3 = 10
25 + 75 = 100   19 + □ = 20

Use near doubles
6 + 7 = double 6 + 1 = 13

Addition using partitioning and recombining
34 + 45 = (30 + 40) + (4 + 5) = 79

Counting on or back in repeated steps of 1, 10, 100, 1000
86 + 57 = 143 (by counting on in tens and then in ones)
460 - 300 = 160 (by counting back in hundreds)

Add the nearest multiple of 10, 100 and 1000 and adjust
24 + 19 = 24 + 20 – 1 = 43
458 + 71 = 458 + 70 + 1 = 529

Use the relationship between addition and subtraction
36 + 19 = 55   19 + 36 = 55
55 – 19 = 36   55 – 36 = 19

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED.
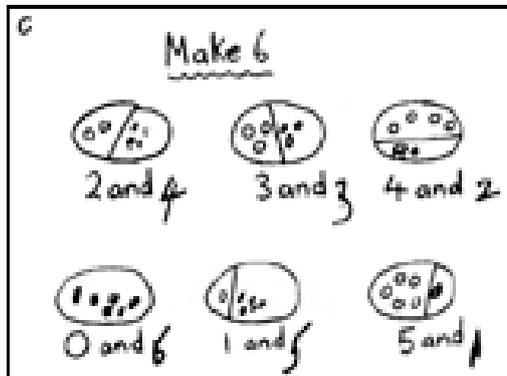*THEY ARE NOT REPLACED BY WRITTEN METHODS.*

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Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.
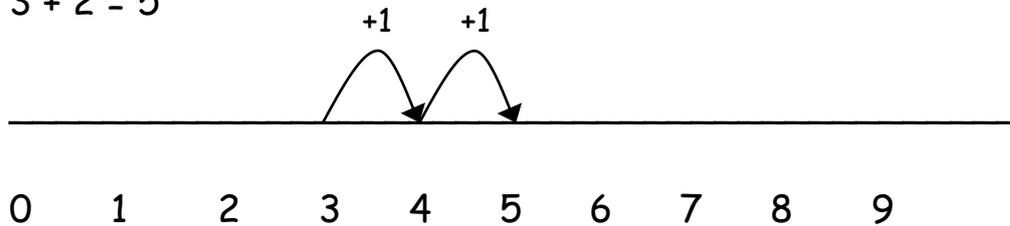
Make 6

2 and 4   3 and 3   4 and 2

0 and 6   1 and 5   5 and 1

The children are taught to remember number bonds using a variety of images and practical resources such as teddy bears, frogs and beads.

🧸🧸🧸🧸 **+** 🧸 **=5**

They use number lines and practical resources to support calculation and teachers *demonstrate* the use of the numberline.

3 + 2 = 5

+1   +1

0   1   2   3   4   5   6   7   8   9

Key vocabulary:
Calculating words:
More, less, the same, many, lots, fewer, greater than, more than, less than.
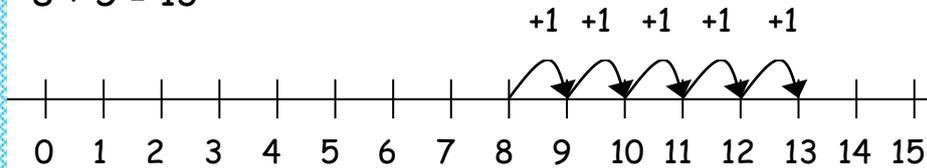*For further guidance and ideas on activities please go to:* www.ncetm.org.uk

ADDITION

Add with numbers up to 20

Children begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

8 + 5 = 13

+1 +1 +1 +1 +1

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

- Have access to a wide range of counting equipment, everyday objects, number tracks and number lines, and be shown numbers in different contexts, e.g. money and measures.
- Read and write addition (+) and equals (=) signs within number sentences.
- Interpret addition number sentences and solve missing box problems using concrete objects and number line addition to solve them: 8 + 3 = ☐
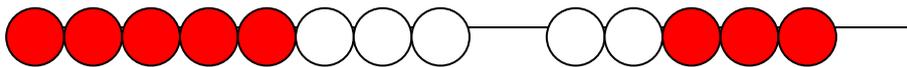 15 + 4 = ☐   5 + 3 + 1 = ☐   ☐ + ☐ = 6

This builds on from prior learning of adding by combining two sets of objects into one group (5 cubes and 3 cubes) in Early Years.

Bead strings or bead bars can be used to illustrate addition including bridging through ten, e.g. calculating 8 + 5 by counting on 2 then counting on 3.
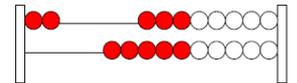
A Bead bar

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line.
Key skills for addition at Yr1:
- Read and write numbers to 100 in numerals including 1 – 20 in words.
- Recall bonds to 10 and 20, and addition facts within 20.
- Count to and across 100.
- Count in multiples of 1, 2, 5, and 10.
- Solve simple 1 – step problems involving addition using objects, number lines and pictorial representations.
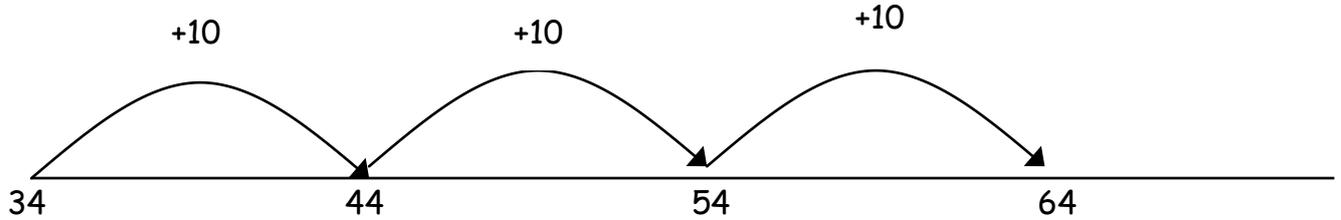
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<u>Y2</u>

Add with 2-digit numbers – developing mental fluency with addition and place value involving 2-digit numbers, then establish more formal methods.

Children will begin to use 'empty number lines' themselves starting with the larger number and counting on, firstly in ones to the next ten and then in tens.

✓ First add 2 digit numbers and tens

34 + 30

✓ Then 2 digit numbers and units: use a range of equipment, hundred squares etc to build confidence and applying number bond knowledge to make the jump to the next ten.

16 + 7
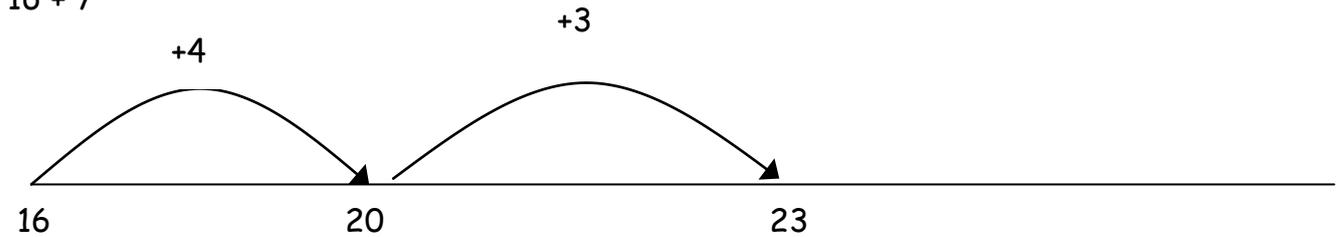
Add pairs of 2 digit numbers, moving to the partitioned column method **ONLY** when secure adding tens and ones:

63 + 16

Partitioned column method:

Step 1: Only provide examples that do **NOT** cross the tens boundary until they are secure with the method itself.

23 + 34   (set out like this)

$$20 + 3$$
$$\underline{30 + 4}$$
$$50 + 7 \quad = 57$$

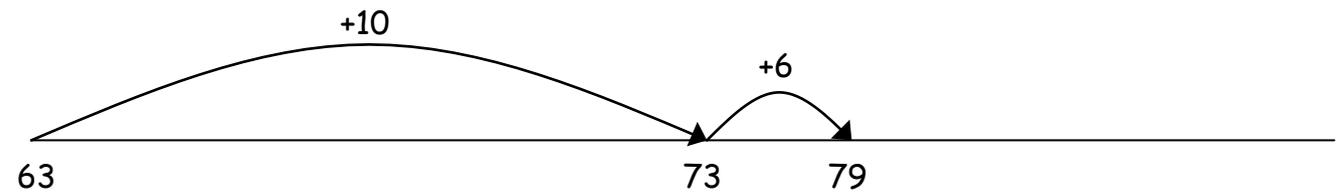**Step 2:** Once children can add a multiple of ten to a 2 digit number mentally, they are ready for adding pairs of 2 digit numbers that **DO** cross the tens boundary.

58 + 43 (set out like this)

$$
\begin{array}{r}
50 + 8 \\
\underline{40 + 3} \\
90 + 11 \quad = 101
\end{array}
$$

To support understanding, pupils may physically carry out the calculation with Base 10 apparatus or place value counters or charts, then compare their practical version to the written form, to help them build on their understanding.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary.

Key skills for addition at Yr 2:

- Add a 2 digit number and tens
- Add a 2 digit number and ones
- Add pairs of 2 digit numbers
- Add three single digit numbers
- Show that adding can be done in any order (commutative law)
- Recall number bonds to 20 and number bonds of tens to 100
- Count in steps of 2, 3 and 5 and count in tens from any number.
- Understand the place value of 2 digit numbers
- Compare and order numbers to 100 using ≤ ≥ and = signs
- Read and write numbers to at least 100 in numerals and words
- Solve problems with addition, using concrete objects, pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.

Y3
Add numbers with up to 3 digits.

The use of a number line will be continued if appropriate with the use of increasingly large numbers. The expanded column addition method will be introduced.

236 + 73   (set out like this)

```
      236
  +    73
  --------
        9
      1 00
      200
  --------
      309
```

In order to carry out this method of addition:
- Children need to recognise the value of the hundreds, tens and ones without recording the partitioning.
- Pupils need to be able to add in columns.

Move to the compact column addition, with carrying:

```
      236
  +    73
  --------
      309
        1
```

Carry numbers below the bottom line

Children who are very secure and confident with 3 digit expanded column addition should be moved onto the compact method, being introduced to 'carrying' for the first time. Compare the two methods as a teaching point to develop understanding of the process and the reduced number of steps.

Remind pupils the actual value is three tens add seven tens and not 'three add seven', and this equals ten tens.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact.
Key skills for addition at Yr 3:
- Read and write numbers to 1000 in numerals and words.
- Add 2 digit numbers mentally, including those exceeding 100.
- Add the following mentally: a 3 digit number with ones, tens or hundreds.
- Estimate answers to calculations using the inverse to check.
- Solve problems including missing number problems using number facts, place value and complex addition.
- Recognise place value of 3 digit numbers.
- Practise mental addition strategies such as number bonds and partitioning.

Add numbers with up to 4 digits.

The move from the expanded method to the compact method is completed with reinforcement about starting with the ones column and carrying underneath the calculation.

Introduce the compact column addition method by asking children to add two given numbers together using the method they are familiar with. Teacher then models the compact method with carrying, asking children to discuss similarities and differences and establish how it is carried out.

3517 + 396 = 3913 (set out like this)

```
  3517
+  396
  3913
   1 1
```

Reinforce correct place value by reminding them the actual value is 5 hundreds add 3 hundreds, NOT 5 add 3, for example.

Carry numbers underneath the bottom line.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact, thousands, hundreds, digits, inverse.

Key skills for addition at Yr 4:
- Select most appropriate method: mental, jottings or written and explain why.
- Recognise the place value of each digit in a four digit number.
- Round any number to the nearest 10, 100 or 1000.
- Estimate and use inverse operation to check answers.
- Solve 2 step problems i8n context, deciding which operations and methods to use and why.
- Find 1000 more or less than a given number.
- Continue to practise a wide range of mental addition strategies, ie number bonds, add the nearest multiple of 10, 100 or 1000 and adjust, use near doubles, partitioning and recombining.
- Add numbers up to 4 digits using the formal written compact column addition.
- Solve 2 step problems in context, deciding which operations and methods to use and why.
- Estimate and use inverse operations to check answers to a calculation.

A D D I T I O N

Add numbers with more than 4 digits (including money, measures and decimals with different numbers of decimal places.

```
    £23.59
  + £  7.55
    £31.14
     1 1 1
```

The decimal point should be aligned in the same way as the other place value columns, and must be in the same column in the answer.

```
    23481
  +  1362
    24843
        1
```

Numbers should exceed 4 digits.

```
    19.01
  + 3.65
    0.70
    23.36
     1 1
```

Pupils should be able to add more than two values, carefully aligning place value columns.

Empty decimal places can be filled with zero to show the place value in each column.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths.

Key skills for addition at Yr 5:
- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies
- ie. add the nearest multiple of 10, 100, 100 and adjust; use near doubles, inverse, partitioning and
- re-combining; using number bonds.
- Use rounding to check answers and accuracy.
- Solve multi-step problems in contexts, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000.
- Add numbers with more than 4 digits using formal written method of columnar addition.

Y6
Add several numbers of increasing complexity

```
    23.361
     9.080
  + 59.770
     1.300
  ─────────
    93.511
     2 1 2
```

```
    81059
     3668
  + 15301
    20551
  ─────────
   120579
    1 1 1 1
```

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths.

Key skills for addition at Yr 6:
- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy.
- Pupils understand how to add mentally with larger numbers and calculations of increasing complexity.

S U B T R A C T I O N

Find 1 more or 1 less from a group of up to 5 objects, then 10 objects.

Children should:
- Use a range of practical activities to explore what one more and one less means.
- Generate one more and one less problems based on their own interests.
- Begin to use the vocabulary (below) to do with adding and subtracting.

Key vocabulary:

Calculating words:

More, less, the same, many, lots, fewer, greater than, more than, less than.

For further guidance and ideas on activities please go to: www.ncetm.org.uk
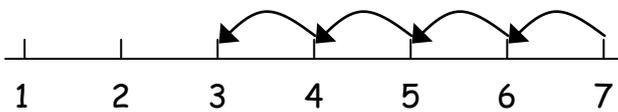

S U B T R A C T I O N

Y1

Subtract from numbers up to 20

Children consolidate understanding of subtraction practically showing subtraction on bead strings, using cubes etc. and in familiar contexts, and are introduced to more formal recording using number lines as below:

Subtract by taking away:

Read, write and interpret number sentences with – and = signs.

Number lines (already drawn)

1   2   3   4   5   6   7

- 7 – 4 = 3
- Start at 7 and count back 4 steps
- Do not count the "7" as a step.

Model subtraction using hundred squares and numbered number lines/ number tracks and practically.

Find the distance between:

This will be introduced practically with the language: 'find the distance between' and 'how many more?' in a range of familiar contexts.

e.g.

Seven is 3 more than four.

e.g. I am 2 years older than my sister

Mental subtraction: Children should start recalling subtraction facts up to and within 10 and 20, and should be able to subtract zero.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_?
Key skills for subtraction at Y1:
- Given a number, say one more or one less.
- Count to and over 100, forward and back, from any number.
- Represent and use subtraction facts to 20 and within 20.
- Subtract with one-digit and two-digit numbers to 20, including zero.
- Solve one-step problems that involve addition and subtraction, using concrete objects (ie bead string, objects, cubes) and pictures, and missing number problems.

SUBTRACTION

Y2
Subtract with 2 digit numbers

Continue with the concept of counting backwards. This is also reinforced as a mental strategy for subtraction. If secure on the use of counting backwards then introduce the concept of complimentary addition to solve subtraction sums using a number line **(Finding the difference)**. Relate to everyday problems such as change in money problems.

11 -7=

> Initially model backwards so children can see the answer is the same. Reinforce subtraction even though going forward by using the vocabulary **difference between**.

0   1   2   3   4   5   6   7   8   9   10   11
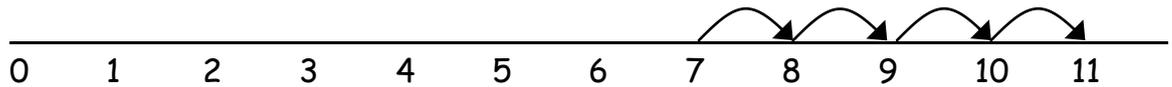
Start at 7 and count on until they reach the larger number. How many steps have they taken?
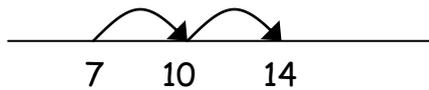
Blank number lines

14 – 7 =

Count on from the **smaller number** then add up the jumps.

+3   +4

3+4= 7

7   10   14

- Always go to the next friendly number (nearest 10)

> Use the inverse and addition knowledge to work it out.

Horizontal reading

30 – 26 = 4

> Link and model on a number line.

Use of empty boxes

☐☐ = 17-13

12 + ☐ = 16

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units

Key skills for subtraction at Y2:
- Recognise the place value of each digit in a two-digit number.
- Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100
- Subtract using concrete objects, pictorial representations, 100 squares and mentally including : 2 digit by 1, 2 digit by 10 and 2 digit by 2.
- Show that subtraction of one number from another is not commutative.
- Recognise and use inverse relationships between + and -, using to check calculations and missing number problems.
- Solve simple + and – problems using concrete objects and applying mental and written methods.
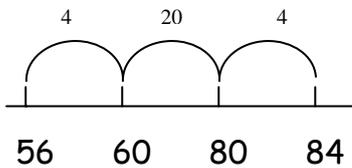- Read and write numbers to at least 100 in numerals and in words.

Y3
Subtracting with 2 and 3 digit numbers.

Complimentary addition (counting from the smaller to the larger)

84 – 56 =

```
      4      20      4
    ⌒      ⌒      ⌒
  56    60    80    84
```

Remember to make the first step to the next ten or hundred before making further jumps. These jumps could be multiples of ten or hundred depending on the size of the numbers being used.

= 4 + 20 + 4 = 28

Continue to reinforce counting on as a mental strategy for close together numbers and also for numbers that are "nearly" multiples of 10, 100, 1000 or £'s, which make it easier to count on. Follow the model above of counting to the next ten first. Reinforces number bonds and 100 square counting.

Move onto partitioned column subtraction <u>only</u> if secure on the above.

Decomposition/Partitioning

Strategy introduced through partitioning the number into tens and ones. All modelling of the method to be done using base 10 equipment and Calculation mats.

Step 1. Introduce the method where no exchange of tens and units is required.
e.g. 89 – 35 = 54

Step 2. Introduce "exchanging" through practical subtraction. Use the calculation mats and base 10 equipment until they are secure
e.g. 72 - 47

Before subtracting 7 from the 72 blocks they will need to exchange a row of 10 for ten ones. Then subtract 7 followed by the 4 tens.

Step 3. Once pupils are secure with the understanding of exchanging, they can use the partitioned column method to subtract 2 and 3 digit numbers.
e.g. 238 – 146 =92   set out as follows:

$$
\begin{array}{r}
\,100 \\
\,\cancel{200} + {}^{1}30 + 8 \\
-\underline{100 + 40 + 6} \\
0 + 90 + 2
\end{array}
$$

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit.

SUBTRACTION

Key skills for subtraction at Y3:
- Subtract mentally a: 3-digit number & ones, 3-digit number & tens, 3-digit number & hundreds .
- Estimate answers and use inverse operations to check.
- Solve problems, including missing number problems.
- Find 10 or 100 more or less than a given number.
- Recognise the place value of each digit in a 3-digit number .
- Counting up differences as a mental strategy when numbers are close together or near multiples of ten.
- Read and write numbers up to 1000 in numerals and words.
- Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or
- 21), and select most appropriate methods to subtract, explaining why.

N.B
An example of the calculation mat is attached to the back of the policy and can be found on the main school system in the maths folder called 'calculation mat'.

S U B T R A C T I O N

Subtracting with up to 4 digit numbers.

Partitioned column subtraction with "exchanging" (decomposition):
As introduced in year 3 but moving towards more complex numbers and values
e.g. 2754 – 1562 = 1192

> Partitioning of 4 digit numbers using place value cards to reinforce exchanging. If not secure return to 3 digit numbers modelled using Base 10 and Calculation mats.

Compact column subtraction

$$\begin{array}{r} 2\,\overset{6}{\cancel{7}}\,{}^{1}5\,4 \\ -\ 1\,5\,6\,2 \\ \hline 1\,1\,9\,2 \end{array}$$

> To introduce the compact method, ask children to perform a subtraction calculation with the familiar partitioned column subtraction then display the compact version for the calculation they have done. Ask pupils to consider how it relates to the method they know, what is similar
> and what is different, to develop an understanding of it.

Mental strategies

A variety of mental strategies must be taught and practised, including counting on to find the difference where numbers are closer together, or where it is easier to count on.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit, inverse.

Key skills for subtraction at year 4:
- Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.
- Children select the most appropriate and efficient methods for given subtraction calculations.
- Estimate and use inverse operations to check answers.
- Solve addition and subtraction 2-step problems, choosing which operations and methods to use and why.
- Solve simple measure and money problems involving fractions and decimals to two decimal places.
- Find 1000 more or less than a given number.
- Count backwards through zero, including negative numbers.
- Recognise place value of each digit in a 4-digit number Round any number to the nearest 10, 100 or 1000
- Solve number and practical problems that involve the above, with increasingly large positive numbers.

Y5
Subtraction with at least 4 digit numbers

Compact column subtraction (with exchanging)

Subtracting with larger integers.

$$
\begin{array}{r}
{}^2\cancel{3}\,{}^1{}^0\cancel{1}\,{}^0\,{}^4\cancel{5}\,{}^1\,6 \\
-\quad 2\,1\,2\,8 \\
\hline
2\,8{,}9\,2\,8
\end{array}
$$

> Children who are still not secure with number facts and place value will need to remain on the partitioned column method until ready for the compact method.

Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

$$
\begin{array}{r}
{}^6\cancel{7}\,{}^1{}^0\cancel{1}\,{}^1\,6\,{}^8\cancel{9}\cdot 0 \\
-\quad\ \ 3\,7\,2\cdot 5 \\
\hline
6\,7\,9\,6\cdot 5
\end{array}
$$

> Add a „zero" in any empty decimal places to aid understanding of what to subtract

Number line method to be used for counting backwards and forwards for negative numbers and for time problems. Number line counting on to be used as a checking of compact method calculations involving decimals – e.g. measures or money.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal.

Key skills for subtraction at year 5:
- Subtract numbers mentally with increasingly large numbers.
- Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy.
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Count forwards or backwards in steps of powers of 10 for any given number up to 1 million.
- Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through 0.
- Round any number up to 1 million to the nearest 10, 100, 1000, 10 000 and 100 000.

Subtraction with increasingly large and more complex numbers and decimal values

Using the compact method to subtract more complex integers.

$$
\begin{array}{r}
^{7}\!\!\!\!\not{1}\ ^{10}\!\!\!\!\not{0}\ {}^{1}5\ .\ ^{3}\!\!\!\!\not{4}\ ^{1}1\ 9\ \text{kg} \\
-\qquad 3\ 6\ .\ 0\ 8\ \text{kg} \\
\hline
6\ 9\ .\ 3\ 3\ 9\ \text{kg}
\end{array}
$$

Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places, remembering to put a zero in as a place holder.

Number line method to be used for counting backwards and forwards for negative numbers and for time problems. Number line counting on to be used as a checking of compact method calculations involving decimals – e.g. measures or money, although pupils should be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting the most appropriate method to work out subtraction problems.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal.
Key skills for subtraction at year 6:
Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
Read, write, order and compare numbers up to 10 million and determine the value of each digit
Round any whole number to a required degree of accuracy
Use negative numbers in context, and calculate intervals across zero.
Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate.

M U L T I P L Y I N G

Multiply with concrete objects, arrays and pictorial representations.

To be used as a means of introducing the idea of grouping for the more able children, on an informal basis. To include written pictorial activities.

e.g.

| 2 | + | 2 | + | 2 |

- Give children experience of counting equal group of objects in 2s,
- 5s and 10s.
- Present practical problem solving activities involving counting equal sets or groups, as above.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count

Key skills for multiplication at Y1:
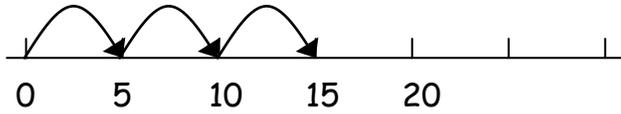- Count in multiples of 2, 5 and 10.
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects,
- pictorial representations and arrays with the support of the teacher.
- Make connections between arrays, number patterns, and counting in twos, fives and tens.
- Begin to understand doubling using concrete objects and pictorial representations.
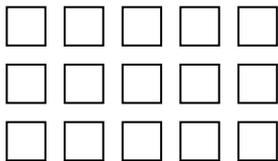

M U L T I P L Y I N G

Y2
Multiplying using arrays and repeated addition (using at least 2s, 5s and 10s)

Use repeated addition on a number line

Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using x and = signs.

5 + 5 + 5 = 3 lots of 5 = 3 x 5 = 15

Array (displayed as boxes or pictorially)

  5 x 3 = 15

3 x 5 = 15

Use arrays to help teach children to understand the commutative law of multiplication, and give examples such as 3 x __ = 6. Use practical apparatus such as bead strings or concrete objects to show groupings.

Mental recall: Children should begin to recall multiplication facts for 2, 5 and 10 times tables through practice in counting and understanding of the operation.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times…

Key skills for multiplication at Y2:
- Count in steps of 2, 3 and 5 from zero, and in 10s from any number.
- Recall and use multiplication facts from the 2, 5 and 10 multiplication tables, including recognising odds and evens.
- Write and calculate number statements using the x and = signs.
- Show that multiplication can be done in any order (commutative).
- Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods, and multiplication facts.
- Pupils use a variety of language to discuss and describe multiplication.

Y3

Multiply 2 digits by a single digit number.

Introduce the grid method for multiplying 2 digits by single digits. Start with a '1' single digit in the tens column e.g. 16 x 4 then extend to multiples of ten.

e.g. 23 x 8 =

x	20	3
8	160	24

Link the layout of the grid to an array initially.

Introduce the grid method with children physically making an array to represent the calculation (e.g. make 8 lots of 23 with 10s and 1s place value counters), then translate this to grid method format.

To do this, children must be able to:
- Partition numbers into tens and units
- Multiply multiples of ten by a single digit (e.g. 20 x 4) using their knowledge of
- multiplication facts and place value
- Recall and work out multiplication facts in the 2, 3, 4, 5, 8 and 10 times tables.
- Work out multiplication facts not known by repeated addition or other taught mental strategies (e.g. commutative law, working out near multiples and adjusting, doubling etc.) Strategies to support this are repeated addition using a number line, bead bars and arrays.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times..., partition, grid method, multiple, product, tens, ones, value.

Key skills for multiplication at Y3:
- Recall and use multiplication facts for the 2, 3, 4, 5, 8 and 10 multiplication tables, and multiply multiples of 10.
- Write and calculate number statements using the multiplication tables they know, including 2-digit x single-digit, drawing upon mental methods, and progressing to reliable written methods.
- Solve multiplication problems, including missing number problems.
- Develop mental strategies using commutativity (e.g. 4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240)
- Solve simple problems in contexts, deciding which operations and methods to use.
- Develop efficient mental methods to solve a range of problems e.g using commutativity (4 × 12 × 5 = 4 × 5 × 12 = 20 × 12 = 240) and for missing number problems   x 5 = 20, 3 x   = 18,   x   = 32

M U L T I P L Y I N G

Multiply 2 and 3 digits by a single digit using all multiplication tables.

Developing the grid method:
e.g. 135 x 5 = 680

x	100	30	5
5	500	150	30

```
  500
+ 150
   30
_____

  6 8 0
```

> Encourage column addition to add accurately.

> Move onto short multiplication (see Y5) if and when children are confident and accurate multiplying 2 and 3-digit numbers by a single digit this way, and are already confident in "carrying" for written addition.

Children should be able to:

- Approximate before they calculate, and make this a regular part of their
- calculating, going back to the approximation to check the reasonableness of their
- answer. e.g: 346 x 9 is approximately 350 x 10 = 3500
- Record an approximation to check the final answer against.
- Multiply multiples of ten and one hundred by a single-digit, using
- their multiplication table knowledge.
- Recall all times tables up to 12 x 12

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times..., partition, grid method, multiple, product, sets of, inverse.

Key skills for multiplication at Y4:

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication facts for all multiplication tables up to 12 x 12.
- Recognise place value of digits in up to 4-digit numbers
- Use place value, known facts and derived facts to multiply mentally, e.g. multiply by 1, 10, 100, by 0, or to multiply 3 numbers.
- Use commutativity and other strategies mentally 3 x 6 = 6 x 3 , 2 x 6 x 5 = 10 x 6 , 39x7 = 30 x 7 + 9 x 7.
- Solve problems with increasingly complex multiplication in a range of contexts.
- Count in multiples of 6, 7, 9, 25 and 1000
- Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)

M U L T I P L Y I N G

Multiply up to 4 digits by 1 or 2 digits.

- Introducing column multiplication:
  Introduce by comparing a grid method calculation to a short method multiplication, to see how the steps are related, but notice how there are less steps involved in the column method..
- Children need to be taught to approximate first e.g. for 72 x 38, they will use rounding: 72 x 38 is approximately 70 x 40 = 2800 and use the approximation to check the reasonableness of their answer against it.

x	300	20	7
4	1200	80	28

→

```
    3 2 7
  x     4
  1 3 0 8
```

> Pupils could be asked to work out a given calculation using the grid, and then compare it to „your" column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps.

Introducing long multiplication for multiplying 2 digits.

e.g. 18 x 13 = 234

	10	8
10	100	80
3	30	24

> 18 x 3 on the 1st row
> (8 x 3 = 24, carrying the 2 for twenty, then 1 x 3). 18 x 10 on the 2nd row. Put a zero in units first stressing it is a place holder, then say 8 x 1, and 1 x 1.

> The grid could be used to introduce long multiplication, as the relationship can be seen in the answers in each row

```
    1 8
  x 1 3
    5 4
  1 8 0
  2 3 4
```

Move towards more complex numbers.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times..., partition, grid method, multiple, product, sets of, inverse, square, factor, integer, decimal, short/long multiplication, carry.

Key skills for multiplication at Y5:
- Identify multiples and factors, using knowledge of multiplication tables to 12x12.
- Solve problems where larger numbers are decomposed into their factors
- Multiply and divide integers and decimals by 10, 100 and 1000
- Recognise and use square and cube numbers and their notation
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.

M U L T I P L Y I N G

Short and long multiplication as in Y5, and multiply decimals with up to 2 decimal places by a single digit.

	3	·	1	9
×			8	
2	5	·	5	2
	₁		₇	

> Line up the decimal points in the question and the answer.

Apply this method to money and other measures.

Children will be able to:
Use rounding and place value to make approximations before calculating and use these to check answers against.
Use short multiplication (see Y5) to multiply numbers with more than 4-digits by a single digit; to multiply money and measures, and to multiply decimals with up to 2d.p. by a single digit.
Use long multiplication (see Y5) to multiply numbers with at least 4 digits by a 2-digit number.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times..., partition, grid method, multiple, product, sets of, inverse, square, factor, integer, decimal, short/long multiplication, carry, tenths, hundredths, decimal.

Key skills for multiplication at Y6:

- Recall multiplication facts for all times tables up to 12 x 12 (as Y4 and Y5).
- Multiply multi-digit numbers, up to 4-digit x 2-digit using long multiplication.
- Perform mental calculations with mixed operations and large numbers.
- Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods.
- Estimate answers using round and approximation and determine levels of accuracy.
- Round any integer to a required degree of accuracy.
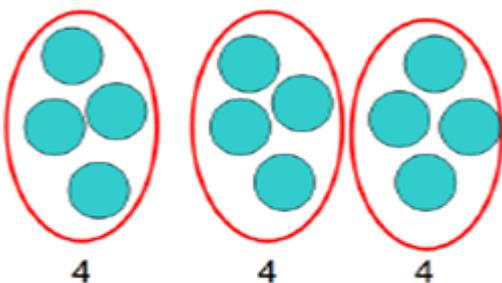
Y1

Group and share small quantities

Using objects, diagrams and pictorial representations to solve problems involving **both grouping and sharing**

Children **MUST** understand the difference between 'sharing' and 'grouping'.

Grouping: How many groups of 4 can be made with 12 stars? = 3

Sharing:

4   4   4

12 shared between 3 is 4

> **Example division problem in a familiar context:**
> There are 6 pupils on this table and there are 18 pieces of fruit to share between us. If we share them equally, how many will we each get?
> Can they work it out and give a division statement… ?
> "18 shared between 6 people gives you 3 each."

Pupils should:
- Know that sharing uses the vocabulary 'one for you, one for me' (see Y2 example)
- Use lots of practical apparatus, arrays and picture representations
- Be taught to understand the difference between 'grouping' objects (How many groups of 2 can you make?) and 'sharing' (Share these sweets between 2 people)
- Be able to count in multiples of 2s, 5s and 10s.
- Find **half** of a set of objects by sharing into 2 equal groups.

Key vocabulary: share, share equally, one each, two each…, group, groups of, lots of, array.

Key number skills needed for division at Y1:

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher
- Through grouping and sharing small quantities, pupils begin to understand, division, and finding simple fractions of objects, numbers and quantities.
- They make connections between arrays, number patterns, and counting in twos, fives and tens.

Y2

Group and share, using the ÷ and = symbols.

Use objects, arrays, diagrams and pictorial representations, and grouping on a number line.

Arrays:
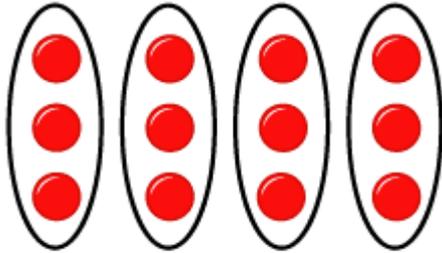
$12 \div 3 = 4$

This represents **12 ÷ 3, posed as** how many groups of 3 are in 12?

Pupils should also show that the same array can represent **12 ÷ 4 = 3** if grouped horizontally.

Know and understand sharing <u>and</u> grouping:

There are 6 sweets, how many people can have 2 sweets each?
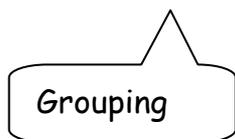
Grouping

Sharing

Children should be taught to recognise whether the problem requires sharing or grouping.

D I V I S I O N

Grouping using a number line:

+3   +3   +3   +3

0 1 2 3 4 5 6 7 8 9 10 11 12
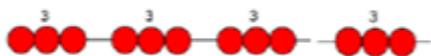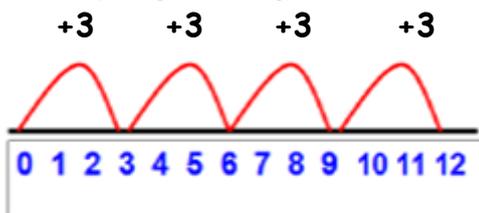
12 ÷ 3 = 4

Group from zero in equal jumps of the divisor to find out "how many groups of _ in _ ?". Pupils could practise grouping using a bead string or practical apparatus to work out problems like - A CD costs £3. How many CDs can I buy with £12?
This is an important method to develop understanding of division as grouping.

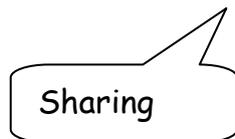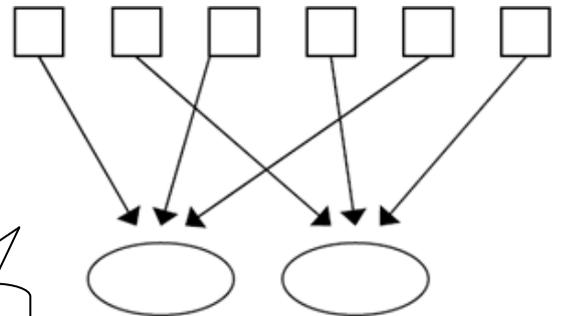**Key vocabulary**: share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over

Key number skills needed for division at Y2:

- Count in steps of 2, 3, and 5 from 0
- Recall and use multiplication and division facts for the **2, 5 and 10** multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the x, ÷ and = signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts,

D I V I S I O N

Y3
Divide 2 digit numbers by a single digit

Grouping on a number line: Initially without remainders and then adding remainders.

$$13 \div 3 = 4 \text{ r } 1$$

+3   +3   +3   +3   r1

0 1 2 3 4 5 6 7 8 9 10 11 12 13
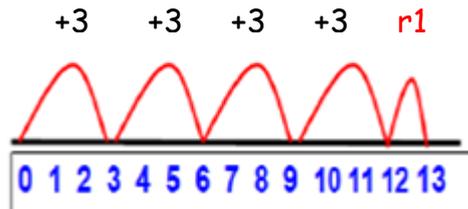
> **Children continue to work out unknown division facts by grouping on a number line from zero.** They are also now taught the concept of **remainders,** as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s and 10s, ready for „carrying‟ remainders across within the short division method.

Children need to:

Continue to use the number line method for increasingly larger 2 digit by 1 digit calculations including remainders (see number line above).

Use times tables facts and be secure on inverse operation as a mental skill and calculation skill for division.

Real life contexts need to be used to gain a full understanding and the ability to recognise the place of division and how to apply it to problems.

Key vocabulary: share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry‟, remainder, multiple

Key number skills needed for division at Y3:

- Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables (through dou-bling, connect the 2, 4 and 8s).
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to for-mal written methods.
- Solve problems, in contexts, and including missing number problems, involving multiplication and division.
- Pupils develop efficient mental methods, for example, using multiplication and division facts (e.g. using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (30 × 2 = 60, so 60 ÷ 3 = 20 and 20 = 60 ÷ 3).
- Pupils develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers

Y4

Divide up to 3 digit numbers by a single digit (without remainders initially)

If children are not ready to move to short division, continue to model division using a number line:

$60 ÷ 5 = 12$ (10+2 lots of 5)
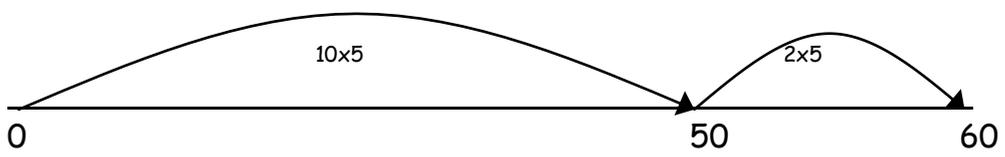
Introduce short division:

STEP 1: Pupils must be secure with the process of short division for dividing 2-digit numbers by a single digit. Children must understand how to calculate remainders, using this to 'carry' remainders within the calculation process. Stress starting on the left, the tens column. Ask how many 4's in 7 because each digit is tackled individually and you always start in the left hand column, in this case the tens column and the answer goes above but still in the tens column. Talk about 'carrying' the remainder.

STEP 2: Pupils move onto dividing numbers with up to **3-digits** by a single digit, however problems and calculations provided should **not result in a final answer with remainder** at this stage. Stress starting on the left, in this case in the hundreds column. The carried remainder becomes the ten of the next digit in the calculation (see example). Children who exceed this expectation may progress to Y5 level.

When the answer for the **first column** is zero (1 ÷ 5, as in example), children could initially write a zero above to acknowledge its place, and must always 'merge' the number with the next digit (5's in 1 becomes 5's in 18) Use a line under the two digits to model this.

Include money and measure contexts when confident as well as other real life contexts.

D I V I S I O N

Key vocabulary: share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, _carry', remainder, multiple, divisible by, factor

Key number skills needed for division at Y4:

Recall multiplication and division facts for all numbers up to 12 x 12.

- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example 200 × 3 = 600 so 600 ÷ 3 = 200
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

YR 5
Divide up to 4 digits by a single digit, including those with remainders.

Short division, including remainder answers:

$$0\ 6\ 6\ 3\ r\ 5$$
$$8\overline{)5\,^53\,^50\,^29}$$

The answer to 5309 ÷ 8 could be expressed as 663 and five eighths, 663 r 5, as a decimal, or rounded as appropriate to the problem involved.

Short division with remainders: Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where **pupils consider the meaning of the remainder and how to express it,** ie. as a fraction, a decimal, or as a rounded number or value , depending upon the context of the problem.

Introduced to long division on a number line: (3 digit by 2 digit)

Initially without remainders
e.g. 357 ÷ 17 = 21

Start by finding groups of the divisor:
10 x 17 = 170   (this list can be added to –
5 x 17 = 85   3 groups, 20 groups)
2 x 17 = 34

| X10 | x10 | x1 |

0   170   340   357
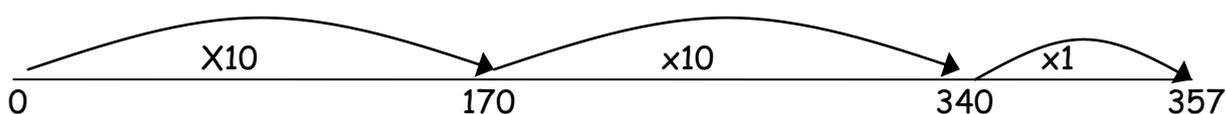
Count up the groups on the number line.

Key vocabulary: share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry', remainder, multiple, divisible by, factor, quotient, prime number, prime factors, composite number (non-prime)

Key number skills needed for division at Y5:

Recall multiplication and division facts for all numbers up to 12 x 12 (as in Y4).

Multiply and divide numbers mentally, drawing upon known facts.

Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number.

Solve problems involving multiplication and division where larger numbers are decomposed into their factors.

Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.

Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.

Work out whether a number up to 100 is prime, and recall prime numbers to 19.

Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

Use multiplication and division as inverses.

Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. 98 ÷ 4 = 24 r 2 = 24.¹/₂= 24.5 ≈ 25).

Solve problems involving combinations of all four operations, including understanding of the equals sign, and including division for scaling by different fractions and problems involving simple rates.

Y6

Divide at least 4 digits by both single-digit and 2-digit numbers
(including decimal numbers and quantities)

Short division, for dividing by a single digit: *e.g. 6497 ÷ 8*

$$8\overline{)6\,^64\,9\,7\cdot\,^10\,^20\,^40}$$
$$0\;8\;1\;2\cdot1\;2\;5$$

Calculating a decimal remainder:
The one remainder is carried onto zeros after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved.

Short division with remainders:
Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

Continue with long division for 3 and 4 digit numbers by a 2 digit number following the number line method shown in Y5. Use real life context to deepen understanding and application. Children should be encouraged to take the biggest possible jumps on the number line and use a column method addition as they go along to ensure the right number of groups will fit into the given number.

Key vocabulary: share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, quotient, prime number, prime factors, composite number (non-prime), common factor.

Key number skills needed for division at Y6:
- Recall and use multiplication and division facts for all numbers to 12 x 12 for more complex calculations
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long divi-sion, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all 4 operations.
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem.
- Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.